

Atmospheric and Solar Neutrino Masses and Abelian Flavor Symmetry

Kiwoon Choi

*Department of Physics, Korea Advanced Institute of Science and Technology
Taejon 305-701, Korea*

Abstract

Recent atmospheric and solar neutrino experiments suggest that neutrinos have small but nonzero masses. They further suggest that mass eigenvalues have certain degree of hierarchical structures, and also some mixing angles are near-maximal while the others are small. We first survey possible explanations for the smallness of neutrino masses. We then discuss some models in which the hierarchical pattern of neutrino masses and mixing angles arises as a consequence of $U(1)$ flavor symmetries which would explain also the hierarchical quark and charged lepton masses.

Typeset using REVTeX

I. INTRODUCTION

Atmospheric and solar neutrino experiments have suggested for a long time that neutrinos oscillate into different flavors, thereby have nonzero masses [1]. In particular, the recent Super-Kamiokande data strongly indicates that the observed deficit of atmospheric muon neutrinos is due to the near-maximal $\nu_\mu \rightarrow \nu_\tau$ oscillation [2]. Solar neutrino results including those of Super-Kamiokande, Homestake, SAGE and GALLEX provide also strong observational basis for $\nu_e \rightarrow \nu_\mu$ or ν_τ oscillation [3].

The minimal framework to accomodate the atmospheric and solar neutrino anomalies is to introduce small but nonzero masses of the three known neutrino species. In the basis in which the charged current weak interactions are flavor-diagonal, the relevant piece of low energy effective lagrangian is given by

$$\overline{e_L} M^e e_R + g W^{-\mu} \overline{e_L} \gamma_\mu \nu_L + \overline{(\nu_L)^c} M^\nu \nu_L, \quad (1)$$

where the 3×3 mass matrices M^e and M^ν are not diagonal in general. Diagonalizing M^e and M^ν ,

$$\begin{aligned} (U^e)^\dagger M^e V^e &= D^e = \text{diag}(m_e, m_\mu, m_\tau), \\ (U^\nu)^T M^\nu U^\nu &= D^\nu = \text{diag}(m_1, m_2, m_3), \end{aligned} \quad (2)$$

one finds the effective lagrangian written in terms of the mass eigenstates

$$\overline{e_L} D^e e_R + W^{-\mu} \overline{e_L} \gamma_\mu U \nu_L + \overline{(\nu_L)^c} D^\nu \nu_L, \quad (3)$$

where the MNS lepton mixing matrix is given by

$$U = (U^e)^\dagger U^\nu. \quad (4)$$

Upon ignoring CP-violating phases, U can be parametrized as

$$\begin{aligned} U &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} \end{pmatrix} \end{aligned} \quad (5)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. Within this parameterization, the mass-square differences for atmospheric and solar neutrino oscillations can be chosen to be

$$\Delta m_{\text{atm}}^2 = m_3^2 - m_2^2, \quad \Delta m_{\text{sol}}^2 = m_2^2 - m_1^2, \quad (6)$$

while the mixing angles are given by

$$\theta_{\text{atm}} = \theta_{23}, \quad \theta_{\text{sol}} = \theta_{12}, \quad \theta_{\text{rea}} = \theta_{13}, \quad (7)$$

where θ_{rec} describes for instance the neutrino oscillation $\nu_\mu \rightarrow \nu_e$ in reactor experiments.

The atmospheric neutrino data suggest near-maximal $\nu_\mu \rightarrow \nu_\tau$ oscillation [2] with

$$\begin{aligned}\Delta m_{\text{atm}}^2 &\sim 3 \times 10^{-3} \text{ eV}^2, \\ \sin^2 2\theta_{\text{atm}} &\sim 1.\end{aligned}\tag{8}$$

As for the solar neutrino anomaly, four different oscillation scenarios are possible [3] though the large mixing angle (LMA) MSW oscillation is favored by the recent Super-Kamiokande data:

$$\begin{aligned}\text{SMA MSW} : \Delta m_{\text{sol}}^2 &\sim 5 \times 10^{-6} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} \sim 5 \times 10^{-3}, \\ \text{LMA MSW} : \Delta m_{\text{sol}}^2 &\sim 2 \times 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} \sim 0.8, \\ \text{LOW MSW} : \Delta m_{\text{sol}}^2 &\sim 10^{-7} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} \sim 1, \\ \text{LMA VAC} : \Delta m_{\text{sol}}^2 &\sim 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta_{\text{sol}} \sim 0.7.\end{aligned}\tag{9}$$

There is in fact an important constraint from reactor experiments, e.g. CHOOZ [4], indicating no ν_μ oscillation into ν_e , thereby leading to

$$4U_{e3}^2 \approx \sin^2 2\theta_{13} \lesssim 0.15\tag{10}$$

Putting the atmospheric and solar neutrino data together while taking into account the CHOOZ constraint, one can consider the following three patterns of neutrino masses and mixing angles:

I. Bi-maximal mixing with LMA MSW solar neutrino oscillation:

$$\begin{aligned}m_2/m_3 &\sim \lambda \text{ or } \lambda^2, \\ (|s_{23}|, |s_{12}|, |s_{13}|) &\sim \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \lambda^k\right),\end{aligned}\tag{11}$$

II. Bi-maximal mixing with LOW MSW or LMA VAC solar neutrino oscillation:

$$\begin{aligned}m_2/m_3 &\sim \lambda^4 \text{ or } \lambda^5, \\ (|s_{23}|, |s_{12}|, |s_{13}|) &\sim \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \lambda^k\right),\end{aligned}\tag{12}$$

III. Single-maximal mixing with SMA MSW solar neutrino oscillation:

$$\begin{aligned}m_2/m_3 &\sim \lambda^2, \\ (|s_{23}|, |s_{12}|, |s_{13}|) &\sim \left(\frac{1}{\sqrt{2}}, \lambda^2, \lambda^k\right),\end{aligned}\tag{13}$$

where $\lambda \equiv \sin \theta_C \sim 0.2$ for the Cabibbo angle θ_C and

$$m_3 \sim 5 \times 10^{-2} \text{ eV}, \quad k \geq 1\tag{14}$$

in all cases. These neutrino results can be compared with the following quark and charged lepton masses and mixing angles:

$$\begin{aligned}(m_t, m_c, m_u) &\sim 180 (1, \lambda^4, \lambda^8) \text{ GeV}, \\ (m_b, m_s, m_d) &\sim 4 (1, \lambda^2, \lambda^4) \text{ GeV}, \\ (m_\tau, m_\mu, m_e) &\sim 1.8 (1, \lambda^2, \lambda^5) \text{ GeV}, \\ (\sin \phi_{23}, \sin \phi_{12}, \sin \phi_{13}) &\sim (\lambda^2, \lambda, \lambda^3),\end{aligned}\tag{15}$$

where ϕ_{ij} denote the mixing angles of the Cabbibo-Kobayashi-Maskawa matrix parametrized as (5).

The above neutrino masses and mixing angles involve some small numbers. Obviously, the most distinctive one is $m_3/m_\tau \sim 3 \times 10^{-10}$, leading to the old question “why neutrinos are so light compared to their charged lepton counterparts?” Though not extremely small as m_3/m_τ , there are other small numbers like m_2/m_3 , $1 - \sin^2 2\theta$ for near-maximal mixing and $\sin^2 2\theta$ for small mixing, which may require some explanations. For instance, for the scenarios I and II, one may wonder how near maximal θ_{23} and small m_2/m_3 can be simultaneously obtained and also how θ_{13} can be made to be small while keeping θ_{23} and θ_{12} near maximal. Similarly, for the scenario III, one can ask what would be the flavor structure yielding small $m_2/m_3 \sim \lambda^2$ and $\theta_{12} \sim \lambda^2$, but near maximal θ_{23} . In this talk, we first survey possible explanations for the smallness of neutrino masses, and then discuss some models in which the hierarchical patterns of neutrino masses and mixing angles arise as a consequence $U(1)$ flavor symmetries [5,6] which would explain also the hierarchical quark and charged lepton masses.

II. WHY NEUTRINOS ARE SO LIGHT?

Here we discuss four possible mechanisms which would suppress the resulting neutrino mass. These mechanisms are not orthogonal to each other, so one can take more than one mechanism in order to make the neutrino mass small enough.

A. Seesaw-type mechanism

In seesaw-type model, neutrinos are light since their masses are induced by the exchange of superheavy particles. At low energies, the effects of such heavy particles are described by the operator

$$\frac{1}{M} LLHH \quad (16)$$

where L and H are the lepton and Higgs doublets, respectively, and M denotes the mass scale of the exchanged heavy particle. This gives a neutrino mass

$$m_\nu \sim \frac{\langle H \rangle \langle H \rangle}{M} \sim 5 \times 10^{-2} \left(\frac{10^{15} \text{ GeV}}{M} \right) \text{ eV} \quad (17)$$

which can be as small as the atmospheric neutrino mass for $M \sim 10^{15} \text{ GeV}$. There are two different ways to generate the above $d = 5$ operator. One is the exchange of superheavy singlet neutrino [7] (Fig. 1) which corresponds to the conventional seesaw mechanism, and the other is the exchange of superheavy triplet Higgs boson [8] (Fig. 2). For the case of singlet neutrino exchange, the underlying lagrangian includes

$$hHLN + M_N NN + h.c., \quad (18)$$

where N is a singlet neutrino with huge Majorana mass M_N . Integrating out N then yields the operator (16) with $M = M_N/h^2$. For the case of triplet Higgs exchange [8,9], one starts from

$$hTLL - \frac{1}{2}M_T^2 TT^* - M'_T TH^* H^* + h.c., \quad (19)$$

where T is a Higgs triplet with huge mass M_T . Again integrating out T leads to (16) with $M = M_T^2/hM'_T$. The seesaw-type mechanism is perhaps the simplest way to get small neutrino mass. However it would be rather difficult to probe other effects of the involved superheavy particles than generating the neutrino mass.

B. Frogatt-Nielsen mechanism

Neutrino mass can be small if the couplings which are responsible for neutrino mass are suppressed by a spontaneously broken (gauge) symmetry by means of the Frogatt-Nielsen mechanism [10]. As an example, consider again a model with singlet neutrino N but now with weak scale $M_N \sim 10^2$ GeV. Suppose that the operator HLN carries a nonzero integer charge n of some discrete or continuous (gauge) symmetry G of the model and G is spontaneously broken by the VEV of a standard model singlet ϕ which has charge -1 . Then HLN in the bare action is forbidden, however there can be higher-dimensional coupling $\phi^n HLN/M_*^n$ allowed by G where M_* corresponds to the fundamental (or UV-cutoff) scale of the model. This results in the effective Yukawa coupling

$$\left(\frac{\langle\phi\rangle}{M_*}\right)^n HLN \equiv \epsilon^n HLN, \quad (20)$$

and so the neutrino mass

$$m_\nu \sim \frac{\epsilon^{2n} \langle H \rangle \langle H \rangle}{M_N} \sim 5 \times 10^{-2} \left(\frac{\epsilon^n}{3 \times 10^{-7}}\right)^2 \left(\frac{10^2 \text{ GeV}}{M_N}\right) \text{ eV}. \quad (21)$$

By choosing appropriate values of n and ϵ , one can easily accommodate the atmospheric neutrino mass even when the singlet neutrino has a weak scale mass.

In supersymmetric models, one can implement the Frogatt-Nielsen mechanism for small neutrino mass without introducing any singlet neutrino. As an example, consider a supersymmetric model with $U(1)$ flavor symmetry whose symmetry breaking order parameter $\epsilon \sim \lambda$ (Cabbibo angle). The $U(1)$ charges of $H_1 H_2$ and $L H_2$ are assumed to be -1 and $-n$, respectively, where $H_{1,2}$ and L denote the two Higgs doublets and the lepton doublet superfields in the MSSM. Then the supergravity Kähler potential can contain

$$K = \frac{\phi^*}{M_*} H_1 H_2 + \left(\frac{\phi^*}{M_*}\right)^n L H_2, \quad (22)$$

while the holomorphy and $U(1)$ do not allow the supergravity superpotential contain a term like $\phi^m H_1 H_2$ or $\phi^m L H_2$. After the spontaneous breaking of supersymmetry and also of $U(1)$, this Kähler potential gives rise to the μ -type terms in the effective superpotential:

$$W_{eff} = \mu H_1 H_2 + \mu' L H_2,$$

where $\mu \sim \epsilon^* m_{3/2} \sim \lambda m_{3/2}$ and $\mu' \sim \epsilon^{*n} m_{3/2} \sim \lambda^n m_{3/2}$. Here the first term in W_{eff} is just the conventional μ -term and the second corresponds to the bilinear R -parity violating term. As is well known, the bilinear R -parity violation leads to the neutrino mass [11]

$$m_\nu \sim \frac{\mu'^2 \langle H_2 \rangle \langle H_2 \rangle}{\mu^2 M_{1/2}} \sim \lambda^{2(n-1)} M_{weak} \quad (23)$$

where the gaugino mass $M_{1/2}$ and μ are assumed to have the weak scale value M_{weak} . This neutrino mass can be of order the atmospheric neutrino mass if $n = 9$, which would be obtained for instance if the $U(1)$ charges of H_1 , H_2 , L are 4, -5 , -4 , respectively.

In fact, supersymmetric models always contain an intrinsically small symmetry breaking parameter, i.e. $m_{3/2}/M_{Planck}$ describing the size of SUSY breaking. If m_ν is suppressed by a symmetry G which is broken by the SUSY breaking dynamics, small m_ν/M_{weak} and $m_{3/2}/M_{Planck}$ have a common dynamical origin. They are then related to each other by the Frogatt-Nielsen mechanism of G , e.g

$$\frac{m_{2/3}}{M_{Planck}} \sim \epsilon^k, \quad \frac{m_\nu}{M_{weak}} \sim \epsilon^l, \quad (24)$$

where ϵ is the symmetry breaking order parameter of G , and k and l are model-dependent integers. If $k/l = 4/3$ and $m_{3/2} \sim M_{weak}$, one obtains $m_\nu/M_{weak} \sim 10^{-12}$ which is the correct value for the atmospheric neutrino mass. $m_{3/2}/M_{Planck}$ and m_ν/M_{weak} is always related to each other when G is a discrete R -symmetry [12] which appears quite often in compactified string theory. It is also possible to relate m_ν/M_{weak} with $m_{3/2}/M_{Planck}$ by means of other type of symmetry [13].

C. Radiative generation of neutrino mass

Even when the neutrino mass is zero at tree level, if the lepton number symmetry is softly broken, there can be small finite radiative corrections to neutrino mass [14]. The resulting neutrino mass is suppressed by the loop factor as well as the (potentially) small Yukawa couplings which are involved in the loop. The most typical example is the Zee model with

$$\mathcal{L} = f H L E^c + f' S^+ L L - A S^+ H H' + \dots,$$

where H and H' are $SU(2)$ -doublet Higgs fields, S^+ is a charged $SU(2)$ -singlet Higgs field, and L and E^c stand for the conventional lepton doublet and anti-lepton singlet. Here we will ignore the flavor indices of couplings for simplicity. It is then easy to see that a nonzero neutrino mass is generated at one-loop (Fig. 3), yielding

$$m_\nu \sim \frac{f' f^2 A}{16\pi^2 m_S^2} \langle H \rangle \langle H' \rangle, \quad (25)$$

where m_S is the mass of S^+ .

It is also possible to construct a model in which m_ν is generated at higher loop order [14]. A typical example is given by

$$\mathcal{L} = f H L E^c + f' S^+ L L + f'' S^{--} E^c E^c - A S^+ S^+ S^{--} + \dots,$$

where S^+ and S^{--} are charged $SU(2)$ -singlet Higgs fields. In this model, nonzero m_ν appears at two-loop (Fig. 4), yielding

$$m_\nu \sim \frac{A f^2 f'^2 f''}{(16\pi^2)^2 m_S^2} \langle H \rangle \langle H \rangle. \quad (26)$$

Note that even when S^+ and S^{--} have weak scale masses, the resulting neutrino mass can be as small as the atmospheric neutrino mass if the Yukawa couplings $f, f', f' \sim 10^{-2}$.

D. Localizing singlet neutrino on the hidden brane

Recently it has been noted by Randall and Sundrum (RS) that the large hierarchy between M_{Planck} and M_{weak} can be achieved by localizing the gravity on a hidden brane [15]. In the RS model, the spacetime is given by a slice of $d = 5$ AdS space with two boundaries. A flat 3-brane with positive tension is sitting on one of these boundaries at $y = 0$, while a negative tension 3-brane is on the other boundary at $y = b$. Massless $d = 4$ graviton mode is localized on the positive tension brane (the hidden brane) while the observable standard model fields are confined in the negative tension brane (the visible brane). Since $d = 4$ gravity is localized on the hidden brane, matter fields on the visible brane naturally have very weak gravitational coupling, so a large disparity between M_{weak} and M_{Planck} .

Attempts have been made to incorporate small neutrino mass in the RS model [16]. The model contains a bulk fermion Ψ and a bulk real scalar Φ with the following orbifold boundary condition:

$$\Psi(-y) = \gamma_5 \Psi(y), \quad \Phi(-y) = -\Phi(y), \quad (27)$$

in addition to the bulk graviton and the standard model fields. The action is given by

$$\begin{aligned} S = & \int d^4x dy \sqrt{-g_{(5)}} \left[\frac{1}{2} M_*^3 R_{(5)} - \Lambda_B - \frac{i}{2} \bar{\Psi} \gamma^A D_A \Psi - f \Phi \bar{\Psi} \Psi - m \bar{\Psi}^C \Psi + \dots \right] \\ & + \int_{y=b} d^4x \sqrt{-g_{(4)}} [-\Lambda_v + \kappa H \bar{\Psi} L + \kappa' H \bar{\Psi}^C L + \frac{h}{M_*} LLHH + \dots] \\ & + \int_{y=0} d^4x \sqrt{-g_{(4)}} [-\Lambda_h + \dots], \end{aligned} \quad (28)$$

where M_* is the 5-dimensional Planck mass, $R_{(5)}$ is the $d = 5$ Ricci scalar, Ψ^C is the charge-conjugation of the $d = 5$ spinor Ψ , and H and L are the Higgs and lepton doublets confined in the visible brane. Here Λ_v and Λ_h denote the visible brane tension and the hidden brane tension, respectively, and we write explicitly also the terms for neutrino mass in the visible brane action.

If the bulk and brane cosmological constants satisfy

$$k \equiv \sqrt{\frac{-\Lambda_B}{6M_*^3}} = \frac{\Lambda_h}{6M_*^3} = -\frac{\Lambda_v}{6M_*^3} \quad (29)$$

the model admits the following form of $d = 4$ Poincare invariant spacetime and the corresponding massless $d = 4$ graviton mode:

$$ds^2 = e^{-2k|y|} (n_{\mu\nu} + h_{\mu\nu}(x)) dx^\mu dx^\nu - dy^2. \quad (0 \leq y \leq b). \quad (30)$$

Obviously $h_{\mu\nu}$ is localized around $y = 0$, so its coupling to the energy momentum tensor at $y = b$ is exponentially suppressed by e^{-2kb} . Equivalently, all dimensionful quantities on the visible brane are rescaled by e^{-2kb} . This results in the standard model mass parameter

$M_{weak}^2 \sim e^{-2kb} M_*^2$, while the $d = 4$ Planck mass is given by $M_{Planck}^2 = M_*^3(1 - e^{-kb})/k$, so an exponentially small ratio

$$\frac{M_{weak}}{M_{Planck}} \sim e^{-kb}. \quad (31)$$

The small ratio m_ν/M_{weak} can be similarly obtained in the model of (28) by localizing the zero mode of Ψ on the hidden brane. To implement this mechanism, we need first the lepton number violating couplings (both in bulk and on branes) to be suppressed enough, for instance h , κ' and m/M_* should be less than 10^{-12} in order for $m_\nu \lesssim 0.1$ eV. This can be easily achieved by imposing a discrete symmetry under which

$$\Psi \rightarrow e^{2\pi i/N} \Psi, \quad L \rightarrow e^{2\pi i/N} L, \quad (32)$$

which would result in

$$h = \kappa' = m = 0. \quad (33)$$

Then the Dirac equation for the zero mode of Ψ is given by

$$\left(\frac{\partial}{\partial y} - 2k + i\gamma_5 f \langle \Phi \rangle \right) \Psi_0 = 0, \quad (34)$$

leading to the following solution

$$e^{-3k|y|/2} \Psi_0 = e^{-(2f\langle\Phi\rangle - k)|y|/2} \eta(x) \quad (35)$$

where η denotes the canonically normalized (in $d = 4$ sense) singlet neutrino mode. On the parameter region with $k < 2f\langle\Phi\rangle$, this mode is localized on the hidden brane. As a result, η has an exponentially small Yukawa coupling with H and L on the visible brane, so an exponentially small Dirac neutrino mass. After the proper rescaling of the involved fields, one finds

$$\frac{m_\nu}{M_{weak}} \sim \kappa e^{-(2f\langle\Phi\rangle - k)b/2} \quad (36)$$

which can be small as 10^{-12} to provide the atmospheric neutrino mass. Note that the small neutrino mass obtained by localizing singlet neutrino on the hidden brane is a Dirac mass, however the current neutrino oscillation experiments do not distinguish the Dirac mass from the Majorana mass.

III. MODELS WITH ABELIAN FLAVOR SYMMETRY

Here we discuss some models in which the hierarchical patterns of the atmospheric and solar neutrino masses and mixing angles are obtained by means of $U(1)$ flavor symmetries. Our discussion will be limited to a specific example for bi-maximal mixing with LMA MSW solar neutrino oscillation and another example for near-maximal atmospheric neutrino oscillation and SMA MSW solar neutrino oscillation.

A. A model for bi-maximal mixing

The neutrino masses and mixing angles for bi-maximal mixing with LMA MSW solar neutrino oscillation are given by

$$\begin{aligned} m_2/m_3 &\sim \lambda \text{ or } \lambda^2, \\ (|s_{23}|, |s_{12}|, |s_{13}|) &\sim \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \lambda^k \right) \quad (k \geq 1). \end{aligned} \quad (37)$$

One issue for this pattern of neutrino masses and mixing angles is how could one obtain small θ_{13} and m_2/m_3 while keeping θ_{23} and θ_{12} near maximal. Comparing the MNS mixing matrix $U = U^{e\dagger}U^\nu$ with the parametrization (5), one easily finds that U automatically has a small θ_{13} with bi-maximal θ_{23} and θ_{12} if U^e has only one large mixing by θ_{23} and also U^ν has only one large mixing by θ_{12} . A form of charged lepton mass matrix which would lead to such U^e is

$$M^e \simeq m_\tau \begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix} \quad (38)$$

where $a \ll 1$. For the neutrino mass matrix, we can consider two different forms leading to such U^ν . One is of pseudo-Dirac type:

$$M^\nu = m_3 \begin{pmatrix} b_1 & a & c \\ a & b_2 & d \\ c & d & 1 \end{pmatrix} \quad (39)$$

with $b_i \ll a$ and $c, d \ll 1$, and the other is the plain large mixing between the 1st and 2nd generations:

$$M^\nu = m_3 \begin{pmatrix} a_1 & a_2 & c \\ a_2 & a_3 & d \\ c & d & 1 \end{pmatrix}, \quad (40)$$

where all a_i are of the same order and $c, d \ll 1$. The neutrino mass matrix is further constrained to reproduce the correct mass ratio $m_2/m_3 \sim \lambda$ or λ^2 . Here we assume that M^ν is induced by the conventional seesaw mechanism in supersymmetric model and explore the possibility that the above mass matrix textures are obtained as a consequence of $U(1)$ flavor symmetries.

It is not so trivial to obtain the mass matrix textures (38), (39), (40) from $U(1)$ flavor symmetries since M^ν needs to have same order of magnitudes for the 1st and 2nd generations while M^e needs different ones. This difficulty becomes more severe if we want to obtain a smaller value of θ_{13} . Among the models of $U(1)$ flavor symmetries, the simplest one would be the case of single anomalous $U(1)$ whose breaking is described by a single order parameter $\epsilon = \langle \phi \rangle / M_* \sim \lambda$. Unfortunately, it turns out that the desired textures can not be obtained in this simplest case. The next simple model would be the case of single non-anomalous $U(1)$ which has two symmetry breaking parameters with opposite $U(1)$ charges:

$$\epsilon = \frac{\langle \phi_+ \rangle}{M_*}, \quad \bar{\epsilon} = \frac{\langle \phi_- \rangle}{M_*},$$

where ϕ_{\pm} has the $U(1)$ charge ± 1 . Note that if the scale of $U(1)$ breaking is much higher than the scale of supersymmetry breaking, vanishing $U(1)$ D -term assures $|\epsilon| = |\bar{\epsilon}|$. One can also consider the case of two $U(1)$'s in which one $U(1)$ is anomalous while the other is non-anomalous. One plausible symmetry breaking pattern in this case is that ϵ and $\bar{\epsilon}$ have the $U(1) \times U(1)$ charges $(-1, -1)$ and $(0, 1)$, respectively. In this case, vanishing D -term of anomalous $U(1)$ leads to $|\epsilon| \sim \lambda$, while that of non-anomalous $U(1)$ gives $|\epsilon| = |\bar{\epsilon}|$. In the below, we will present a simple example for each case which gives rise to the mass matrix textures for (37).

Let us first consider the case of single $U(1)$ with two symmetry breaking parameters ϵ and $\bar{\epsilon}$. We will assume that $|\epsilon| = |\bar{\epsilon}| \sim \lambda$. The light neutrino mass matrix which is obtained by the seesaw mechanism is given by

$$M^{\nu} = M^D (M^M)^{-1} (M^D)^T, \quad (41)$$

where M^D is the 3×3 Dirac mass matrix and M^M is the 3×3 Majorana mass matrix of superheavy singlet neutrinos. Let small letters denote the $U(1)$ charges of the capital lettered superfields, e.g. l_i for the lepton doublets L_i , e_i for the anti-lepton singlets E_i^c , n_i for the superheavy singlet neutrinos N_i . Then for the charge assignments of

$$l_i = (2, -2, 0), \quad n_i = (2, -2, 0), \quad e_i = (1, 5, 5), \quad h_1 = h_2 = 0, \quad (42)$$

one finds the mass matrices [17] which are given by

$$M^{\nu} = m_3 \begin{pmatrix} \lambda^4 & A & \lambda^2 \\ A & \lambda^4 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}, \quad M^e = \langle H_1 \rangle \begin{pmatrix} \lambda^7 & \lambda^7 & \lambda^3 \\ \lambda^3 & \lambda^3 & \lambda \\ \lambda^5 & \lambda^5 & \lambda \end{pmatrix} \quad (43)$$

where A is of order one, but does not exceed 1. This mass matrix provides the near bi-maximal θ_{23} and θ_{12} , and also

$$\theta_{13} \sim \lambda^2, \quad \Delta m_{\text{atm}}^2 \sim (1 - A^2) m_3^2, \quad \Delta m_{\text{sol}}^2 \sim \lambda^4 A^2 m_3^2, \quad (44)$$

which can accomodate all experimental data for reasonable values of A and m_3 .

The desired forms of mass matrices can be obtained for the case of $U(1) \times U(1)$ also. If one $U(1)$ is anomalous while the other is non-anomalous, which is the case that appears quite often in compactified string theory, it is quite plausible that $U(1) \times U(1)$ are broken by the two symmetry breaking parameters ϵ and $\bar{\epsilon}$ with the $U(1) \times U(1)$ charges $(-1, -1)$ and $(0, 1)$. In this case, $|\epsilon| = |\bar{\epsilon}|$ and they are naturally of order the Cabbibo angle λ . Then for the charge assignment

$$\begin{aligned} n_1 &= (0, -1), & n_2 &= (0, 1), & n_3 &= (0, 0), \\ l_1 &= (0, -1), & l_2 &= (1, 2), & l_3 &= (0, 0), \end{aligned}$$

one easily finds

$$M^{\nu} \sim m_3 \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 0 & 0 \\ \lambda & 0 & 1 \end{pmatrix} \quad (45)$$

which yields

$$\theta_{13} \sim \lambda, \quad \Delta m_{\text{sol}}^2 \sim \lambda^2 \Delta m_{\text{atm}}^2. \quad (46)$$

B. A model for large atmospheric and small solar neutrino mixings

The neutrino masses and mixing angles for large atmospheric and small solar neutrino mixings are given by

$$m_2/m_3 \sim \lambda^2, \\ (|s_{23}|, |s_{12}|, |s_{13}|) \sim \left(\frac{1}{\sqrt{2}}, \lambda^2, \lambda^k \right) \quad (k \geq 1). \quad (47)$$

The issues for this pattern would be how could one obtain small m_2/m_3 even when θ_{23} is near maximal, and also what would be the reason for small θ_{12} and θ_{13} . Here we present a supersymmetric model with $U(1)$ flavor symmetry in which such pattern of neutrino masses and mixing angles arises naturally.

The model under consideration is the MSSM with R -parity breaking couplings which are suppressed by an anomalous $U(1)$ flavor symmetry with $\epsilon \sim \lambda$ [18]. The most general $SU(3)_c \times SU(2)_L \times U(1)_Y$ -invariant superpotential of the MSSM superfields includes the following lepton number (L) and R -parity violating terms:

$$\Delta W = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c \quad (48)$$

where (L_i, E_i^c) and (Q_i, U_i^c, D_i^c) denote the lepton and the quark superfields, respectively. Another L and R -parity violating term $\mu'_i L_i H_2$ in the superpotential can always be rotated away by a unitary rotation of superfields. Soft SUSY breaking terms also contain the L and R -parity violating terms:

$$\Delta V_{\text{soft}} = m_{L_i H_1}^2 L_i H_1^* + B_i L_i H_2 + C_{ijk} L_i L_j E_k^c + C'_{ijk} L_i Q_j D_k^c, \quad (49)$$

where now all field variables denote the scalar components of the corresponding superfields. In the basis in which $\mu'_i L_i H_2$ in the superpotential are rotated away, non-vanishing B_i and $m_{L_i H_1}^2$ result in the tree-level neutrino mass [11]

$$(M^\nu)_{ij}^{\text{tree}} \approx \frac{g_a^2 \langle \tilde{\nu}_i^* \rangle \langle \tilde{\nu}_j^* \rangle}{M_{1/2}}, \quad (50)$$

where $M_{1/2}$ denote the $SU(2)_L \times U(1)_Y$ gaugino masses and the sneutrino VEV's are given by

$$\langle \tilde{\nu}_i^* \rangle \approx \frac{2M_Z(m_{L_i H_1}^2 \cos \beta + B_i \sin \beta)}{m_{\tilde{l}}^2 + \frac{1}{2}M_Z^2 \cos 2\beta}, \quad (51)$$

where $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$, M_Z is the Z -boson mass, and $m_{\tilde{l}}$ is the slepton soft mass which is assumed to be (approximately) flavor-independent. There are also additional neutrino masses arising from various one-loop graphs involving the squark or slepton exchange [19].

Let the small letters q_i, u_i , e.t.c. denote the $U(1)$ charges of the superfields Q_i, U_i^c , e.t.c. If all L and R -parity violating couplings are suppressed by some powers of λ as is determined

by the $U(1)$ charges of the corresponding operators, the resulting neutrino mass matrix takes the form:

$$(M^\nu)_{ij} = m_3 \begin{pmatrix} \lambda^{2l_{13}} A_{11} & \lambda^{l_{13}+l_{23}} A_{12} & \lambda^{l_{13}} A_{13} \\ \lambda^{l_{13}+l_{23}} A_{12} & \lambda^{2l_{23}} A_{22} & \lambda^{l_{23}} A_{23} \\ \lambda^{l_{13}} A_{13} & \lambda^{l_{23}} A_{23} & A_{33} \end{pmatrix}, \quad (52)$$

where m_3 is the largest mass eigenvalue, $l_{i3} = l_i - l_3$, and all A_{ij} are of order unity. It is then straightforward to see that the near maximal θ_{23} requires $l_2 = l_3$, while the small $\theta_{12} \sim \lambda^2$ requires $l_1 = l_2 + 2$. This eventually leads to the MNS mixing matrix of the form:

$$U \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}. \quad (53)$$

which gives $\theta_{13} \sim \lambda^2$.

So far, we could get the mixing angle pattern $\theta_{23} \sim 1$, $\theta_{12} \sim \theta_{13} \sim \lambda^2$ just by assuming the $U(1)$ charge relations:

$$l_1 - 2 = l_2 = l_3.$$

Still we need to get the mass hierarchies $m_2/m_3 \sim 4 \times 10^{-2}$ and also $m_3/M_{weak} \sim 10^{-12}$. In the model under consideration, $U(1)$ flavor symmetry assures that R -parity violating couplings are all suppressed by $\lambda^{l_i+h_2}$ or $\lambda^{l_i-h_1}$ compared to their R -parity conserving counterparts. As a result, m_3 from R -parity violation obeys roughly

$$m_3/M_{weak} \sim \lambda^{2(l_3+h_2)} \quad \text{or} \quad \lambda^{2(l_3-h_1)}.$$

So if the $U(1)$ charges are arranged to have

$$l_3 + h_2 = l_3 - h_1 = 7 \quad \text{or} \quad 8,$$

the resulting m_3 naturally fits into the atmospheric neutrino mass scale.

One may wonder how m_2/m_3 can be as small as 4×10^{-2} even when the 2nd and 3rd neutrinos mix maximally. The neutrino mass matrix (52) from R -parity violation automatically realizes such unusual scenario since it is dominated by the tree mass (50) which is a rank one matrix. Then the largest mass m_3 is from the tree contribution while m_2 is from the loops, so

$$m_2/m_3 \sim \text{LOOP/TREE}$$

independently of the value of θ_{23} . In fact, we need to make this loop to tree ratio a bit bigger than the generic value in order to get the correct mass ratio $m_2/m_3 \approx 4 \times 10^{-2}$. This is difficult to be achieved within the framework of high scale SUSY breaking, e.g. gravity-mediated SUSY breaking models, while it can be easily done in gauge-mediated SUSY breaking models with relatively low messenger scale. In gauge-mediated SUSY breaking models [20], B_i and $m_{L_i H_1}^2$ can be simultaneously rotated away as μ'_i at the messenger scale M_m , i.e. $B_i(M_m) = m_{L_i H_1}^2(M_m) = 0$ in the basis of $\mu'_i = 0$, and their low energy values at M_{weak} are determined by the RG evolution. Then the tree mass (50) which is determined by these RG-induced B_i and $m_{L_i H_1}^2$ can be made smaller by taking a lower value of the messenger scale.

Soft parameters in gauge-mediated models [20] typically satisfy: $M_a/\alpha_a \approx m_{\tilde{q}}/\alpha_3 \approx m_{\tilde{l}}/\alpha_{1,2}$ at M_m where M_a , $m_{\tilde{q}}$, and $m_{\tilde{l}}$ denote the gaugino, squark and slepton masses, respectively, and $\alpha_a = g_a^2/4\pi$ for the standard model gauge coupling constants. The size of the bilinear term BH_1H_2 in the scalar potential depends upon how μ is generated. An attractive possibility is $B(M_m) = 0$ for which all CP-violating phases in soft parameters at M_{weak} are automatically small enough to avoid a too large electric dipole moment [21]. In this case, the RG-induced low energy value of B yields a large $\tan\beta \approx (m_{H_1}^2 + m_{H_2}^2 + 2\mu^2)/B(M_Z) = 40 \sim 60$.

Analyzing the neutrino masses from R -parity violating couplings which are determined by the RG evolution with the boundary conditions that trilinear soft scalar couplings, B , B_i and $m_{L_i H_1}^2$ are all vanishing at M_m , and also $M_a/\alpha_a \approx m_{\tilde{q}}/\alpha_3 \approx m_{\tilde{l}}/\alpha_{1,2}$ at M_m , one finds [18]

$$(M^\nu)_{ij}^{\text{tree}} \approx 10^{-1} t^4 a_i a_j \left(\frac{\mu^2 M_Z^2}{m_{\tilde{l}}^3} \right), \quad (54)$$

where $a_i = y_b \lambda'_{i33} \sim \lambda^{l_i - h_1}$ for the b -quark Yukawa coupling y_b and $t = \ln(M_m/m_{\tilde{l}})/\ln(10^3)$. The loop mass is given by [18]

$$(M^\nu)_{ij}^{\text{loop}} \approx 10^{-2} t^2 y_b y_\tau^3 \lambda'_{333} (\delta_{i3} \lambda_{j33} + \delta_{j3} \lambda_{i33}) \left(\frac{\mu^2 M_Z^2}{m_{\tilde{l}}^3} \right), \quad (55)$$

where y_τ is the τ -lepton Yukawa coupling and the smaller contributions are ignored. These tree and loop masses then give the following mass hierarchies:

$$\begin{aligned} m_3/M_{weak} &\approx [U_{i3}(M^\nu)_{ij}^{\text{tree}} U_{j3}]/M_{weak} \sim 10^{-1} \lambda^{2(l_3 - h_1)}, \\ m_2/m_3 &\approx [U_{i2}(M^\nu)_{ij}^{\text{loop}} U_{j2}]/m_3 \sim (\text{LOOP/TREE}), \\ m_1/m_2 &\approx [U_{i1}(M^\nu)_{ij}^{\text{loop}} U_{j1}]/m_2 \sim \lambda^4, \end{aligned} \quad (56)$$

where $(\text{LOOP/TREE}) = 10^{-2} (\lambda_{233}/\lambda'_{233}) (\ln 10^3 / \ln \frac{M_m}{m_{\tilde{l}}})^2$, and we have used $\tan\beta \sim 50$, $m_{\tilde{l}} \approx 300 \text{ GeV}$ and $\mu \approx 2m_{\tilde{l}}$ which has been suggested to be the best parameter range for correct electroweak symmetry breaking [21]. To summarize, in this model, small m_2/m_3 is due to the loop to tree mass ratio, while the other small mass ratios m_1/m_2 and m_3/M_{weak} are from $U(1)$ flavor symmetry.

Acknowledgments: I thank E. J. Chun and K. Hwang for useful discussions, and also Y. Kim for drawing the figures. This work is supported by the BK21 project of the Ministry of Education, KRF Grant No. 2000-015-DP0080, KOSEF Grant No. 2000-1-11100-001-1, and KOSEF through the CHEP of KNU.

REFERENCES

- [1] For a recent global analysis, see M. C. Gonzalez-Garcia *et al.*, hep-ph/0009350.
- [2] Y. Fukuda *et al.*, Phys. Rev. Lett. **82** 2644 (1999); Phys. Lett. **B467**, 185 (1999).
- [3] J. Bahcall, P. Krastev and A. Smirnov, hep-ph/0006078.
- [4] CHOOZ Collaboration, Phys. Lett. **B420**, 397 (1998); Phys. Lett. **B466**, 415 (1999).
- [5] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. **B398**, 319 (1993); *ibid*, **B420**, 468 (1994); L. Ib'anez and G. G. Ross, Phys. Lett. **B332**, 100 (1993); P. Binetruy and P. Ramond, Phys. Lett. **B350**, 49 (1995); V. Jain and R. Shrock, Phys. Lett. **B352**, 83 (1995); E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. **B356**, 45 (1995); P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. **B477**, 353 (1996). E. J. Chun and A. Lukas, Phys. Lett. B **387**, 99 (1996); K. Choi, E. J. Chun, and H. Kim, Phys. Lett. B **394**, 89 (1997); Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. **B396**, 150 (1997).
- [6] N. Irges, S. Lavignac and P. Ramond, Phys. Rev. **D58**, 035003 (1998); P. Binetruy, E. Dudas, S. Lavignac and S. A. Savoy, Phys. Lett. **B422**, 171 (1998); J. Ellis, S. Lola and G. G. Ross, Nucl. Phys. **B526**, 115 (1998); C. D. Froggatt, M. Gibson and H. B. Nielson, hep-ph/9811265; Y. Nir and Y. Shadmi, JHEP **9905**, 023 (1999); S. F. King, Nucl. Phys. **B562**, 57 (1999); Nucl. Phys. **B576**, 85 (2000); J. Feng, Y. Nir and Y. Shadmi, Phys. Rev. **D61**, 113005 (2000); M. Berger and K. Siyeon, hep-ph/0010245; Q. Shafi and Z. Tavartkiladze, hep-ph/9904249; hep-ph/9905202; hep-ph/0002150;
- [7] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity* (ed. P. van Nieuwenhuizen and D. Freedman, North-Holland, Amsterdam, 1979) p. 315; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (ed. O. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba, Japan, 1979).
- [8] E. Ma and Utpal Sarkar, Phys. Rev. Lett. **80**, 5716 (1998); E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).
- [9] K. Choi and A. Santamaria, Phys. Lett. **B267**, 504 (1991).
- [10] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
- [11] L. Hall and Suzuki, Nucl. Phys. **B231**, 419 (1984); I.-H. Lee, Phys. Lett. **138B**, 121 (1984).
- [12] K. Choi, E. J. Chun and H. Kim, Phys. Rev. **D55**, 7010 (1997).
- [13] N. Arkani-Hamed *et al.*, hep-ph/0006312; F. Borzumati and Y. Nomura, hep-ph/0007018.
- [14] A. Zee, Phys. Lett. **B93**, 389 (1980); Phys. Lett. **B161**, 141 (1985); K. S. Babu, Phys. Lett. **B203**, 132 (1988); Y. Okamoto and M. Yasue, Phys. Lett. **B466**, 267 (1999); D. Chang and A. Zee, Phys. Rev. **D61**, 071303 (1999); T. Kitabayashi and M. Yasue, Phys. Lett. **B490**, 236 (2000).
- [15] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); Phys. Rev. Lett. **83**, 4690 (1999).
- [16] Y. Grossman and M. Neubert, Phys. Lett. **B474**, 361 (2000).
- [17] Y. Nir and Y. Shadmi, JHEP **9905**, 023 (1999).
- [18] K. Choi, E. J. Chun and K. Hwang, Phys. Rev. **D60**, 031301 (1999).
- [19] For recent works, see E. J. Chun and S. K. Kang, Phys. Rev. **D61**, 075012 (2000); S. Davidson and M. Losada, JHEP **0005**, 021 (2000); O. C. W. Kong, hep-ph/0102113.
- [20] For a review, see G. F. Giudice and R. Rattazzi, Phys. Rep. **322**, 419 (1999).
- [21] R. Rattazzi and U. Sarid, Nucl. Phys. **B501**, 297 (1997).

FIGURES

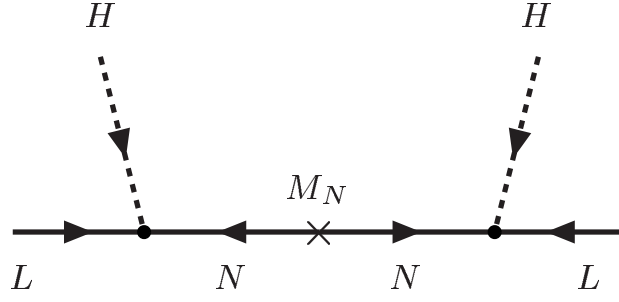


FIG. 1. Small neutrino mass from the exchange of superheavy singlet neutrino

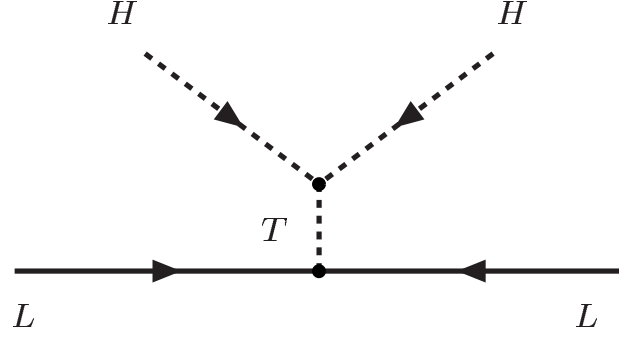


FIG. 2. Small neutrino mass from the exchange of superheavy triplet Higgs

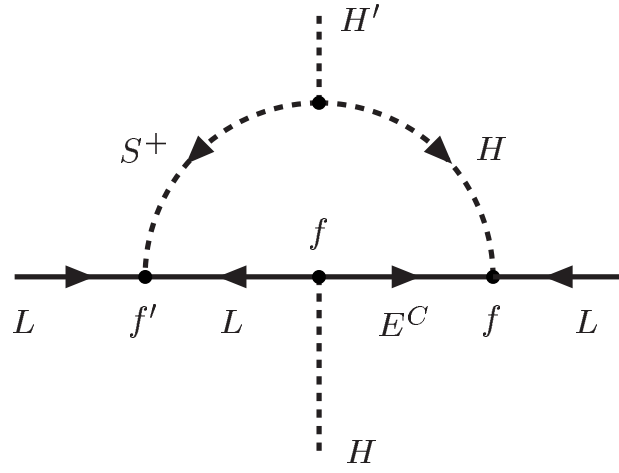


FIG. 3. One-loop neutrino mass in Zee-type model

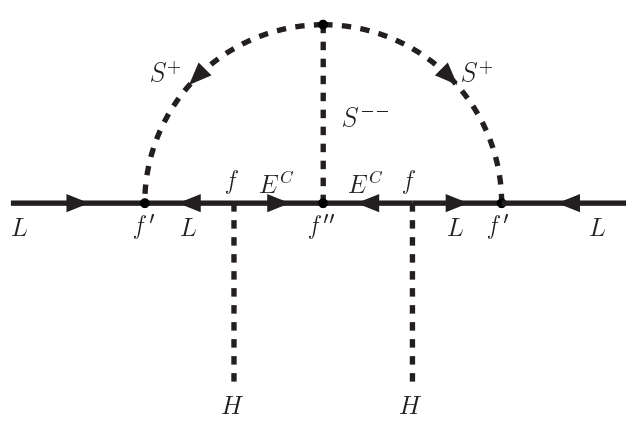


FIG. 4. Two-loop neutrino mass in the variant of Zee model